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STRESS ANALYSIS OF WOUND MAGNETIC TAPE

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A contrast of the existing solution techniques to obtain the stress field generated during the winding of magnetic tape is presented. An experimental technique to measure the wound tape's radial Young's modulus is discussed, and, using the analytical tools developed, a prediction of the stress field in a typically wound reel of magnetic tape is made. The predicted radial pressure is verified experimentally using a pull-tab test. Using this analysis, guidelines are given on the selection of tape geometry, as well as hub geometry and material, to minimize tape defects caused by adverse winding stresses. An analysis of the environmental stresses created in the wound tape by changes in humidity and temperature is described, as well as its application to hub-material selection to reduce slippage in the layers of tape.

INTRODUCTION

Over the past quarter century, several articles have appeared in the literature (*1*)–(*9*) analyzing the stresses generated during the winding of thin flexible webs, such as magnetic tape, film, and paper. A knowledge of these initial stresses, how they can be altered, and how they produce defects in the wound material provide design and winding guidelines that help minimize the adverse effects of such stresses.

During the tape's winding process, when tape is wound on a hub of some compliance and with a certain winding tension, a stress field develops. The most important components of this field are the hoop (also called circumferential) stress, which is caused by the winding tension applied over the cross-sectional area of the tape, and the radial interlayer pressure, which is caused by the radial component of the winding stress. As more and more wraps are added, the radial stress in the inner wraps accumulates continuously. The hub compliance relieves the hoop stress, since the radial displacement of the tape is smaller and so is the hoop strain. At the same time, it reduces the buildup of the radial pressure. However, if the hub is too rigid, the tensile hoop stress generated during winding remains in the wound material near the hub. For materials that have across-the-width variations in mechanical properties, such as magnetic tape, film, or paper, tension bands or lanes result (*9*)–(*11*). With time, these deformations become anelastic and never fully recover (*10*). This process is accelerated at higher temperatures and humidities. The effects of the Poisson's ratio, on the other hand, tend to reduce the tensile hoop stress with increasing radial pressure. Increasing hub compliance reduces the hoop strain in the inner wraps, thereby reducing the hoop stress. If the hub is too compliant, its function is transferred to the wound tape, which produces com-

NOMENCLATURE

- E_r = Young's modulus in the radial direction of the wound tape in compression, which is assumed to be constant throughout the tape (GPa).
- E_θ = Young's modulus in the circumferential direction of the wound tape, in tension, which is assumed to be constant throughout the tape and identical to the Young's modulus of the tape in tension (GPa).
- $\nu_{r\theta}$ = Poisson's ratio in the radial direction of the wound tape, that is, the ratio of the circumferential strain to the radial strain for an element under pure radial stress. It is assumed constant throughout the tape.
- $\nu_{\theta r}$ = Poisson's ratio of the tape in the circumferential direction, that is, the ratio of the radial strain to the circumferential strain for an element under pure circumferential stress. It is assumed constant throughout the tape.
- E_h = Young's modulus of the hub, taken as an isotropic body (GPa)
- ν_h = Poisson's ratio of the hub, taken as an isotropic body
- t_h = hub thickness (mm)
- a = inner hub radius (mm)
- b = outer hub radius (mm)
- c = outer wrap radius (mm)
- R = normalized outer-wrap radius for a fully wound tape; it is c divided by b
- N = total number of wraps in a fully wound tape
- t = wrap thickness (mm)
- w = tape width (mm)
- T_w = winding stress, which is the winding tension divided by the tape's cross-sectional area, wt (GPa)

pressive circumferential stresses in the tape. Sufficient, negative, hoop stresses can cause the wound tape to buckle. Furthermore, the radial pressure during winding must be sufficient to prevent interlayer tape slippage on acceleration. To ensure that the wound tape remains undistorted during storage, the hub must be of uniform compliance, which requires as close a uniform hub thickness as possible. A mismatch in the coefficient of thermal or hygroscopic expansion between the hub and the tape will lead to further stresses induced in the wound tape when there is a temperature or humidity change. These stresses must be considered when selecting hub material and geometry.

The first section of this paper discusses the assumptions made in the formulation of the winding problem. In the literature, two solution techniques exist, one proposed initially by Gutterman in (*2*), which was subsequently used by many others (*3*)–(*8*), and another proposed by Tramposch (*1*). These are contrasted, evaluated, and their respective advantages briefly discussed. A hub compliance is proposed and then the agreement of both approaches is verified. An experimental technique to determine Young's modulus of a wound tape in the radial direction is suggested in the second section, and, using this measured value, a comparison of the predicted and measured radial stresses in a reel of tape is presented. The next section contains an application of our analysis to the selection of hub material and geometry, as well as to the winding tension profile. This analysis is further used to calculate the maximum amount of tape that can be stored on a given hub without adversely affecting the wound tape.

We also include an analysis of the thermal stresses caused by the mismatch in the coefficients of thermal expansion between the hub and the tape, and between the radial and circumferential directions in the tape (*4*), (*12*), (*13*). Centrifugally induced stresses are not discussed here, but are discussed in Refs. (*6*) and (*14*).

ASSUMPTIONS, PROBLEM FORMULATIONS, AND SOLUTION TECHNIQUES

This section contrasts the existing solution techniques, verifies their agreement, and briefly mentions their respective merits.

As mentioned in the introduction, the solution approach proposed by Gutterman has been used by other researchers. We will take the one presented by Altmann (*7*) as typical of this approach and contrast it with that of Tramposch (*1*).

Assumptions

The assumptions made by several researchers differ only in generality, so we list the most general here:

1. The hub is a right-circular cylinder and remains so during and after winding. The relationship between the hub deflection and the radial pressure exerted on it by the tape is linear.
2. The tape is of uniform thickness, which is small in comparison with its width, and offers no resistance to bending.
3. During and following winding, the tape reel is considered to be a homogeneous and orthotropic elastic cylinder. Young's modulus in the radial direction, E_r, is smaller than that in the circumferential direction, E_θ. This is due to the softening effect of the tape's coating and the entrapped air during winding.
4. Although the winding process is continuous, it can be modeled as the successive addition of closed rings with a known internal tension.
5. Shear stresses and relative motion between the layers can be neglected.
6. The stress components at a point in the tape are independent of its width and those in the width direction are negligible, therefore, plane-stress conditions exist.

The assumptions have thus reduced the formulation to a one-dimensional, plane-stress axisymmetric problem, with variations occurring only in the radial direction. We now need to determine the radial- and hoop-stress components at any radial location in the wound tape.

In the remainder of this section, we will use the following notation to refer to the equations shown in Tables 1–3: 1A-3 refers to Table 1, Altmann column, Eq. [3]; 1T-2 refers to Table 1, Tramposch column, Eq. [2]; and so on. The tables show the equations in tabular form for easy comparison and quick reference.

Problem Formulation—Altmann

Consider a hub, whose outer radius is b (see Fig. 1), onto which tape is wound. With the addition of each wrap, wound under tension, there is a change induced in the stress state existing in the already wound wraps. Altmann evaluates the incremental change in the stress state induced at an inner radial location r (normalized with respect to b, $r = \bar{r}/b$, where $\bar{r}$ is the actual radial location), when the current outer wrap is added [see Fig. 1(a)]. At equilibrium, the stress component increments σ_r and σ_θ, namely, the radial and hoop stress, are related through the stress equation of equilibrium 1A-1. Then, using the stress-displacement relationship for an orthotropic, linearly elastic solid under plane-stress conditions 1A-2, he obtains the displacement equation of equilibrium 1A-3. This gives the change in radial displacement u (again normalized with respect to b) at a radial location r caused by the addition of the current outer wrap at a (normalized) radial location s; u is fully determined when two boundary conditions are specified, one at the hub and the other at the current outer wrap. The boundary condition at the hub, 2A-1, relates the radial deflection of the outer hub radius ($r = 1$) to the pressure exerted on it by the wound tape. We have defined E_c, in 2A-1, as the hub elasticity (*8*), for a hub whose geometry is thick, hollow, right-circular cylindrical and whose composition is homogeneous, isotropic, and linearly elastic (*6*), (*15*). E_c is a function of the hub stiffness and geometry. For orthotropic hubs, used by some manufacturers, Lekhnitskii (*16*) has a corresponding formula. The current outer-wrap boundary condition, 2A-2, is a statement of equilibrium for the applied radial pressure, which is caused by the addition of that wrap, of incremental thickness, ds, with internal hoop stress, T_w, caused by the winding tension at a (normalized) radial location s. Note that a variable, winding-tension profile can

Table 1—Problem Formulation	
Altmann	**Tramposch**
Stress equation of equilibrium	
$r\frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$ [1]	$T_{i,j} + P_{i,j} + (r_i + t)\frac{P_{i+1,j} - P_{i,j}}{t} = 0$ [1]
Stress-displacement relationship	
$\sigma_r = \frac{E_r}{(1-\nu_{r\theta}\nu_{\theta r})}\left(\frac{du}{dr} + \nu_{\theta r}\frac{u}{r}\right)$ $\sigma_\theta = \frac{E_\theta}{(1-\nu_{r\theta}\nu_{\theta r})}\left(\frac{u}{r} + \nu_{r\theta}\frac{du}{dr}\right)$ [2]	$P_{i,j} = \frac{-E_r}{1-\nu_{r\theta}\nu_{\theta r}}\left(\frac{u_{i+1,j} - u_{i,j}}{t} + \nu_{\theta r}\frac{u_{i,j}}{r_i}\right)$ $T_{i,j} = \frac{E_\theta}{1-\nu_{r\theta}\nu_{\theta r}}\left(\frac{u_{i,j}}{r_i} + \nu_{r\theta}\frac{u_{i+1,j} - u_{i,j}}{t}\right)$ [2]
Displacement equation of equilibrium	
$\frac{d^2u}{dr^2} + \left(1+\nu_{\theta r} - \nu_{r\theta}\frac{E_\theta}{E_r}\right)\frac{1}{r}\frac{du}{dr} - \frac{E_\theta}{E_r}\frac{u}{r^2} = 0$ [3]	$\frac{u_{i+2,j} - 2u_{i+1,j} + u_{i,j}}{t^2}$ $+\left(1+\nu_{\theta r} - \nu_{r\theta}\frac{E_\theta}{E_r}\right)\frac{1}{r_i}\frac{u_{i+1,j} - u_{i,j}}{t} - \frac{E_\theta}{E_r}\frac{u_{i,j}}{r_i^2} = 0$ [3] $i = 1,\ldots,j - 1$

Table 2—Boundary Conditions	
Altmann	**Tramposch**
At the hub	
$u(1) = \frac{\sigma_r(1)}{E_c}$ [1] Where $E_c = \frac{\left(1 - \frac{a^2}{b^2}\right)E_h}{(1+\nu_h)\frac{a^2}{b^2} + (1-\nu_h)}$ E_h . . . Hub elasticity	$u_{1j} = f\,p_{1j}$ [1] Where $f = \frac{b}{E_c}$ f . . . Hub compliance
At the "current" outer wrap	
$\sigma_r(s) = \frac{T_w}{s}\,ds$ [2] Note: $\sigma_r\,(s+ds) = 0$ See Fig. 1(a)	$P_{j,j} = \frac{T_{j,j}t}{r_j}$ [2] Note: $P_{j+1,j} = 0$ See Fig. 1(b)

be incorporated into the analysis by considering T_w as a function of s in this boundary condition.

Solution Technique

An exact solution of the displacement equation of equilibrium 1A-3, which is a second-order, ordinary, differential equation, is given in 3A-1, with two unknown constants of integration. From the hub compliance's boundary condition 2A-2, we obtain, using 3A-1 and 1A-2, one equation, 3A-2, that relates the unknown constants A and B; from the current outer-wrap boundary condition 2A-2, we get a second equation, 3A-3, again using 3A-1 and 1A-2, that relates A and B. From these two equations, 3A-1 and 3A-2, we can fully determine u and, using 1A-2, also σ_r and σ_θ. The final stress distribution in the fully wound tape, at a radial location r, is calculated by an integration over all the increments added on that radial location to give the total radial and circumferential stress $P(r)$ and $T(r)$, respectively, in 3A-4. There, R is the (normalized) outer-wrap radius of the fully wound tape reel (see Fig. 2).

Problem Formulation—Tramposch

Unlike Altman, Tramposch in his analysis assumes that

$$\nu_{r\theta} = \nu_{\theta r} \, (E_r/E_\theta) \quad . \qquad [1]$$

In the plane-stress, axisymmetric, wound tape, we have two Young's moduli, E_θ and E_r. E_θ is the Young's modulus of the magnetic tape in the circumferential direction, in tension, and E_r is the Young's modulus in the radial direction, in compression. And two Poisson's ratios, $\nu_{r\theta}$ and $\nu_{\theta r}$. McCullough (*17*) comments that these four material constants cannot all be independent and they require that Eq. [1] hold. This assumption is used in Refs. (*1*) and (*3*), but not generally. A measurement of $\nu_{\theta r} = 0.23$ is presented by Bogy et al (*18*). A measured value of $\nu_{r\theta} = 0.05$ is presented by Umanskii et al (*3*), who then use Eq. [1] to get $\nu_{\theta r}$. Those who do not accept Eq. [1], use

$$\nu_{r\theta} = \nu_{\theta r} \quad , \qquad [2]$$

for which there appears to be no theoretical or experimental justification.

While we presented Altmann's analysis in the more general case, we present Tramposch's analysis, as he did, with Eq. [1] holding. Moreover, we do not use the normalized radial distance and radial displacement, as in the previous analysis. Tramposch, with a view to incorporating linear variations in the width direction of tape thickness, hub diameter, and hub flexibility, obtains a discretized (on the wrap thickness) stress equation of equilibrium, 1T-1, relating the induced increment in radial pressure, $T_{i,j}$, and hoop stress, $P_{i,j}$, in the i^{th} wrap caused by the addition of the j^{th} or the current outer wrap. Using the discretized stress-displacement relationship 1T-2, he obtains a three-point, finite-difference, displacement equation of equilibrium, 1T-3, relating the displacements produced in three adjacent inner wraps, caused by the addition of the j^{th} wrap, which is the current outer wrap. The boundary conditions are again the effect of hub compliance, 2T-1, where we define f, the hub compliance, for the same cylindrical hub described in Altmann's analysis, and the current outer-wrap pressure, 2T-2. Again note that a variable, winding-tension profile can be incorporated into this analysis by considering $T_{j,j}$, the winding stress in the current outer wrap, as a function of j in this boundary condition.

Solution Techniques

Rewriting the displacement equation of equilibrium, 1T-3, in the form given in 3T-1, the hub's boundary condition 2T-1, as given in 3T-2 using 1T-2, and the current outer-wrap boundary condition 2T-2, as given in 3T-3, again using 1T-2, produces a matrix equation, 3T-4, for the radial displacements $u_{i,j}$, $1 \leqslant i \leqslant j + 1$, in the wound wraps, which can be solved algebraically. Recall at this stage that $u_{j+1,j}$ is defined as the displacement of the outer surface of the current outer wrap, as illustrated in Fig. 1(b). This approach differs slightly from that of Tramposch's. The solution for $u_{i,j}$ is given in 3T-5. A summation over all the wraps wound on top of a specific wrap gives the total displacement for that wrap. Using the stress-displacement relationship 1T-2, Tramposch obtains the change in stress in the i^{th} wrap caused by the addition of the j^{th} wrap. A summation over all the wraps wound on a specific wrap gives the final stress distribution in that wrap, for each wrap in the fully wound reel of tape, 3T-6.

TABLE 3—PROBLEM SOLUTION

ALTMANN	TRAMPOSCH
Displacement equation of equilibrium gives	
$u(r) = Ar^{\gamma-\delta} + Br^{-(\gamma+\delta)}$	$-a_i u_{i-1,j} + D_i u_{i,j} - u_{i+1,j} = 0 \quad ; i = 2, \ldots, j \quad . \qquad [1]$
Where	Where
$\beta^2 = \dfrac{E_\theta}{E_r}$	$a_i = 1 - \left(1 + \dfrac{E_\theta}{E_r} \dfrac{\bar{t}}{1 + (i-2)\,\bar{t}}\right) \dfrac{\bar{t}}{1 + (i-2)\,\bar{t}} \quad , \bar{t} = \dfrac{t}{b}$
$\delta = \dfrac{1}{2}\left(\nu_{\theta r} - \nu_{r\theta} \dfrac{E_\theta}{E_r}\right) \quad , \gamma = \sqrt{\delta^2 + \beta^2}$	$D_i = 2 - \dfrac{\bar{t}}{1 + (i-2)\,\bar{t}} \quad .$
Hub boundary condition gives	
$A + B = \dfrac{1}{E_r}\,[a_r A + b_r B] \qquad [2]$	$D_1 u_{1,j} - u_{2,j} = 0 \qquad [2]$
Where	Where
$A_r = (\nu_{\theta r} + \eta) \dfrac{E_r}{(1 - \nu_{r\theta}\,\nu_{\theta r})} \quad , b_r = (\nu_{\theta r} - \rho) \dfrac{E_r}{(1 - \nu_{r\theta}\,\nu_{\theta r})}$	$D_1 = 1 - \left(\nu_h - \dfrac{1}{F}\right) \bar{t}$
$\eta = \gamma - \delta \quad , \rho = \gamma + \delta$	$F = \dfrac{f E_r}{1 - \nu_{r\theta}\,\nu_{\theta r}} \dfrac{1}{b}$

TABLE 3—PROBLEM SOLUTION (CONTINUED)

ALTMANN	TRAMPOSCH
"Current" outer wrap boundary condition gives	
$$[a_r A s^{\eta} + b_r B s^{-\rho}]\frac{1}{s} = \frac{-T_w}{s}\,ds \qquad [3]$$	$$-a_{j+1}\,u_{j,j} + D_{j+1}\,u_{j+1,j} = b_{j+1} \qquad [3]$$ Where $$a_{j+1} = 1 - \nu_h \frac{\bar{t}}{1+(j-1)\bar{t}}\ , D_{j+1} = 1$$ $$b_{j+1} = -\left(\frac{1-\nu_{r\theta}\,\nu_{\theta r}}{E_r}\right)\frac{t\,\bar{t}}{1+(j-1)\bar{t}}\,T_{j,j}$$
Solving for A and B gives $$\sigma_r(r) = \frac{1+mr^{-2\gamma}}{r^{1-\eta}}\cdot\frac{s^{1-\eta}}{(1+ms^{-2\gamma})}\cdot\frac{T_w}{s}\,ds \qquad [4]$$ $$\sigma_\theta(r) = \frac{-(\eta-\rho\, mr^{-2\gamma})}{r^{1-\eta}}\cdot\frac{s^{1-\eta}}{(1+ms^{-2\gamma})}\cdot\frac{T_w}{s}\,ds$$ Where $$m = \frac{\gamma+\mu-\dfrac{E_\theta}{E_c}}{\gamma+\mu+\dfrac{E_\theta}{E_c}}\ , \mu = \frac{1}{2}\left(\nu_{\theta r} + \nu_{r\theta}\frac{E_\theta}{E_r}\right)$$	This Gives a Matrix Equation for $u_{i,j}$, $1<i<j+1$ $$\begin{bmatrix} D_1 & -1 & 0 & 0\cdots\cdots & 0 & 0 \\ -a_2 & D_2 & -1 & 0 & \cdot & \cdot \\ 0 & -a_3 & D_3 & -1 & \cdot & \cdot \\ \vdots & & & & \vdots & \vdots \\ \cdot & & & & D_j & -1 \\ 0 & 0 & 0 & 0\cdots\cdots & -a_{j+1} & D_{j+1} \end{bmatrix} \begin{Bmatrix} u_{1,j} \\ \vdots \\ u_{j,j} \\ u_{j+1,j} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ b_{j+1} \end{Bmatrix} \qquad [4]$$ Solve simply $$u_{j+1,j} = \frac{b_{j+1}}{D_{j+1} - a_{j+1}\,s_j} \equiv z_{j+1}$$ For $i = j, j-1, \ldots .\ 1$ $$u_{i,j} = s_i\,u_{i+1}\quad ;\ s_k = \frac{1}{D_k - a_k\,s_{k-1}} \qquad [5]$$ $$k = 2, \ldots, j$$
The total radial and hoop stress distribution at a radial location r in the fully wound reel is,	The total radial and hoop stress distribution in the ith wrap for a total of N wraps wound is,
$$P(r) = \frac{1+mr^{-2\gamma}}{r^{1-\eta}}\int_r^R \frac{s^{1-\eta}}{1+ms^{-2\gamma}}\frac{T_w}{s}\,ds \qquad [5]$$ $$T(r) = T_w - \frac{(\eta-\rho\, mr^{-2\gamma})}{r^{1-\eta}}\int_r^R \frac{s^{1-\eta}}{1+ms^{-2\gamma}}\frac{T_w}{s}\,ds$$	$$P_{i,N} = P_{i,i} - \sum_{j=i+1}^{N}\left[\frac{u_{i+1,j}-u_{i,j}}{t} + \nu_{\theta r}\frac{u_{i,j}}{r_i}\right] \qquad [6]$$ $$T_{i,N} = T_{i,i} - \sum_{j=i+1}^{N}\left[\nu_{\theta r}\frac{u_{i+1,j}-u_{i,j}}{t} + \frac{E_\theta}{E_r}\frac{u_{i,j}}{r_i}\right]$$

Agreement

Thus, under the stipulated conditions, we can evaluate the initial stress conditions in tape wound on a reel. The above analyses have been programmed and both are found to agree. An immediate implication is that, since Altmann's model does not involve tape thickness but only the amount of tape stored, which is tape thickness multiplied by the number of wraps, tape thickness does not affect the initial stress field. This fact is borne out using Tramposch's model. From a programming point of view, Altmann's analysis is represented by a few lines of code involving numerical integration, while that of Tramposch, even though it is only algebraic, is more complicated. Tramposch does, however, present an analysis to account for the across-the-width variations mentioned earlier, which offers reasonable extendibility to his approach.

On completion of the winding of a wrap, the internal

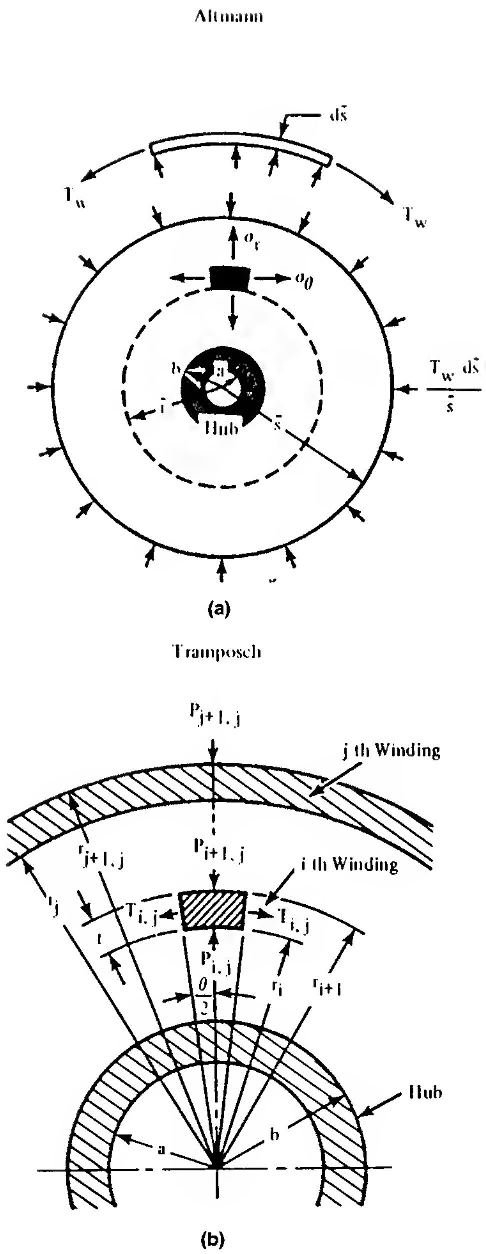

Fig. 1—Altmann and Tramposch models

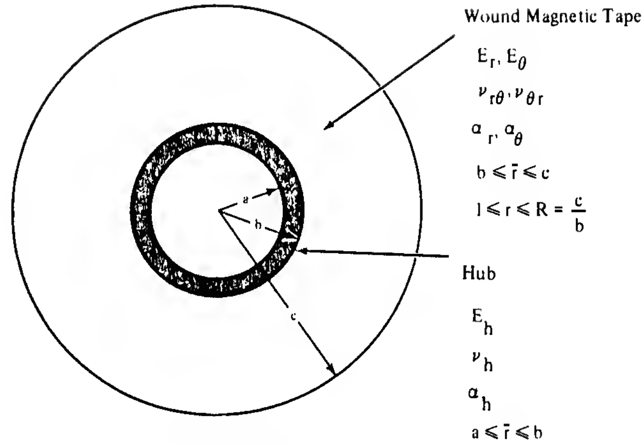

Fig. 2—Fully wound tape geometry

stresses begin to relax, because of the viscoelastic nature of the magnetic tape (*12*), (*19*), (*20*). This process is accelerated at higher temperatures and humidities. Before presenting our predictions of the model, we must obtain the materials' properties needed for its use. This question is addressed in the next section.

EXPERIMENTAL RESULTS

Of the input needed for the analysis presented in the last section, the numerical value for the Young's modulus of the wound tape in the radial direction is the most uncertain. For magnetic tape, two values appear in the literature. In Refs. (*3*) and (*13*), a value of 1.1 GPa is presented, while in Refs. (*4*) and (*6*) a value of 0.17 GPa is given. The value of the radial modulus is dependent to a large extent on the amount of air entrapped during winding. The height of the air film $h(0)$ at the nip during winding is given by Block et al (*21*) as

$$h(0) = 2.233s(\mu Uw/T)^{2/3} \quad , \qquad [3]$$

where U is the winding speed
μ is the viscosity of air
T is the winding tension
s is the current outer-wrap radius
w is the tape width.

The air-film height entrapped increases with winding speed, tape width, and current winding radius, while it decreases with winding tension. Therefore, we expect for increased $h(0)$ a reduced E_r. This is verified experimentally in Ref. (*6*), where the effect of the winding speed on radial pressure in wound tape is observed. A reduced radial pressure was found for an increased winding speed, which follows from a reduced radial modulus. The effect of a reduced radial modulus on the radial pressure is discussed later under "Analysis Application."

The following technique was used to measure E_r. A reel of magnetic tape was placed in a compression apparatus and the wound pack was compressed using a rigid, flat, cylindrical indentor of circular cross section of small radius. The measured load deflection, recorded on a chart recorder, was linear. Since the indentor radius was small, we approximated the tape's cylindrical surface as flat and estimated the Young's modulus using the formula for a flat, circular indentor on an isotropic elastic half space, as

$$d = 4qa(1-\nu_{r\theta}^2)/E_r\pi \qquad [4]$$

where d is the deflection
q is the load
a is the radius of the indentor
E_r is the radial Young's modulus of the tape
$\nu_{r\theta}$ is the radial Poisson's ratio.

Even though $\nu_{r\theta}$ is unknown, the value of E_r is not very sensitive to a variation in a physically reasonable value of $\nu_{r\theta}$. Note that this formula is approximate. A finite element analysis of an indentor on a semi-infinite strip of orthotropic material showed that this formula underestimated the radial modulus by ten percent. Neglecting curvature has a lesser

effect. Typical values we measured were in the range

$$E_r = 0.17 \text{ to } 0.28 \text{ GPa} \qquad [5]$$

which was compatible with that found in Ref. (*6*). We consistently found that the lower value was obtained immediately after winding, while the higher value was obtained from a tape that had been wound for a day or so. This we explain as entrapped air between the layers seeping out. After a day, E_r decreased with time because the relaxation process then dominates. Pfeiffer (*22*) shows that for paper, E_r is proportional to the pressure in the wound roll. This assumption, though more realistic, adds to the complexity of the mathematical problem (*8*).

Then, in an effort to measure the radial-stress distribution in wound tape, we performed the following experiment. Tabs of magnetic tape were inserted periodically during tape winding. Once wound, these tabs were pulled out of the reel. We determined the interlayer pressure from the coefficient of friction and the pull force (*3*), (*6*), (*10*). An experiment to measure the hoop stress inside rolls of paper by splicing an especially made strain gauge directly into the web of paper is described by Hussain et al (*23*). We have not seen or attempted such an experiment on magnetic tape.

To compare the theoretical and experimental results, we ran Altmann's model under the conditions of both Eqs. [1] and [2], with E_r in the range given in Eq. [5], and with

$$\nu_{\theta r} = 0.3, \qquad [6]$$

together with the geometry of the experimental tape reel. A comparison of the predicted radial stress and that measured experimentally is given in Fig. 3. Much better agreement is obtained with Eq. [1] and we have used this.

ANALYSIS APPLICATION

Equipped with our analytical technique and the necessary materials' properties, we can look at the effect of the various variables, such as pack geometry, hub geometry and stiffness, and the winding tension profile, on the stresses generated during winding, and how they can be altered to minimize the probability of creating a stress profile that leads to magnetic tape defects. In the numerical examples we present

a =	50.8 mm	b =	69.85 mm	c =	125.73 mm
E_θ =	3.45 GPa	$\nu_{\theta r}$ =	0.3	E_r =	0.17 GPa
E_h =	6.89 GPa	ν_h =	0.33	t_h =	19.05 mm

together with Eq. [1] holding, since these properties resemble those of common, magnetic tape-reels found today. Then we show the influence of tape-reel geometry, as well as hub geometry and material property variations on the tape's stresses.

The outer hub radius, b, must be increased to permit an increase in the amount of tape stored. This is so because the inner wraps of tape, acting as a hub, go into compression circumferentially as more wraps are added. This effect can be seen in Fig. 4, where we look at the effect of storing more tape on our selected hub. If we increase R, we increase the radial stress as expected, but we also put the inner wraps into compression circumferentially. Thus, long lengths of tape are stored on larger diameter hubs.

The hub should be of uniform stiffness across its width (*24*). Otherwise, it will cause nonuniform collapse along the width, producing a tension gradient in the tape and a nonuniform permanent distortion across the width of the tape with time. Our analysis assumes a hub of uniform stiffness.

Hub thickness and material contribute to the hub's compliance. As mentioned in the introduction, if the hub is too compliant, it shifts its function to the inner wraps and puts them into compression circumferentially. If this compression becomes so high as to overcome the friction between the layers, the tape will fold back on itself and buckle. To avoid this, we must increase the hub's stiffness by increasing its thickness or changing to a material with a higher Young's modulus. When compression occurs farther out in the pack, then we must increase the outer hub's radius or decrease the amount of tape in the pack, as mentioned earlier.

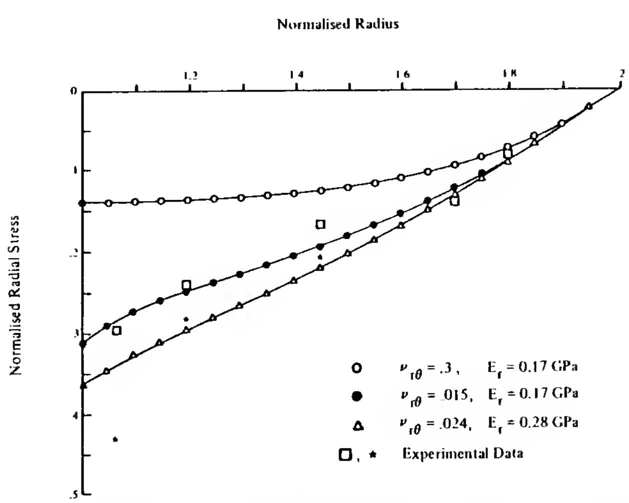

Fig. 3—Comparison of the theoretical and experimental radial stress in two experimental tape reels. Comparison of the effect of the value of Poisson's ratio $\nu_{r\theta}$ on the predicted radial stress. The analysis uses $\nu_{\theta r}$ = 0.3 and E_θ = 3.45 GPa.

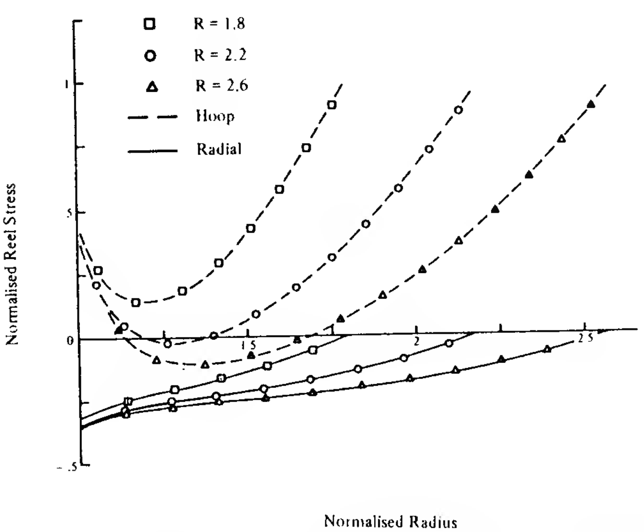

Fig. 4—Predicted effect on the tape's circumferential and radial stresses of the fully wound tape's outer wrap radius.

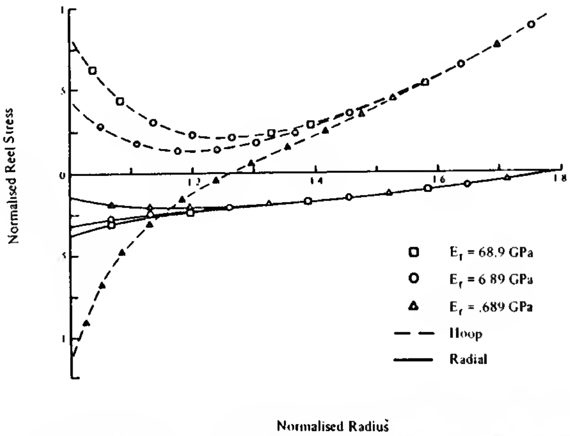

Fig. 5—Predicted effect on the tape's circumferential and radial stresses of the Young's modulus of the hub.

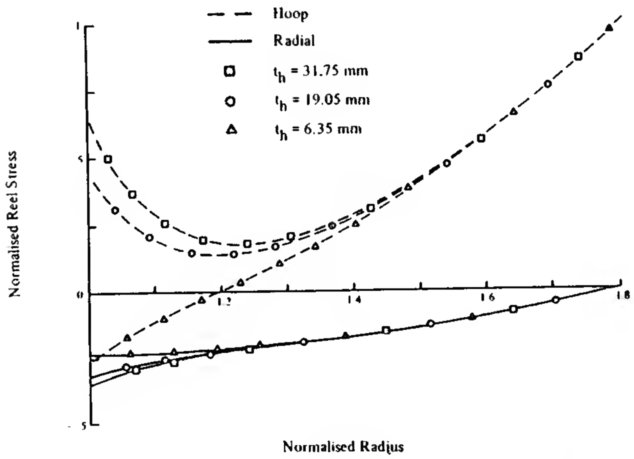

Fig. 6—Predicted effect on the tape's circumferential and radial stresses of the thickness of the hub.

If the hub is too stiff, the high, tensile hoop stresses caused by winding remain in the tape near the hub. This goes unnoticed in narrow tape, but in wide tape, which has significant variations in mechanical properties across its width, these high hoop stresses produce tension bands or lanes in the tape. With time, as the stresses relax, these distortions become permanent and adversely affect the tape's performance. This effect has also been found in paper (*11*), (*12*) and cellophane (*10*). Figure 5 shows the effect on the tape's hoop and radial stresses of a hub that is: too compliant, E_r = 0.689 GPa; too stiff, E_r = 68.9 GPa; and intermediate, E_r = 6.89 GPa. Figure 6 shows the same effect for different hub thicknesses, that is, too compliant, t_h = 6.35 mm; reasonable, t_h = 19.05 mm; and more rigid, t_h = 31.75 mm. To avoid tension ridges, we advise not to use a too rigid hub, thereby keeping the amplitude of the tension ridges small and protecting the tape from more severe distortions.

For a constant winding tension, we see from 3A-5 that the resultant stress is linearly proportional to the winding stress. For wide rolls, it is common to use decreasing winding tension with increasing tape winding radius. The effect on the hoop-stress profile of a linearly decreasing (or tapered) winding tension, which reduces to 0.6 times its initial value during tape winding is shown in Fig. 7.

As mentioned earlier, changing the winding tension and

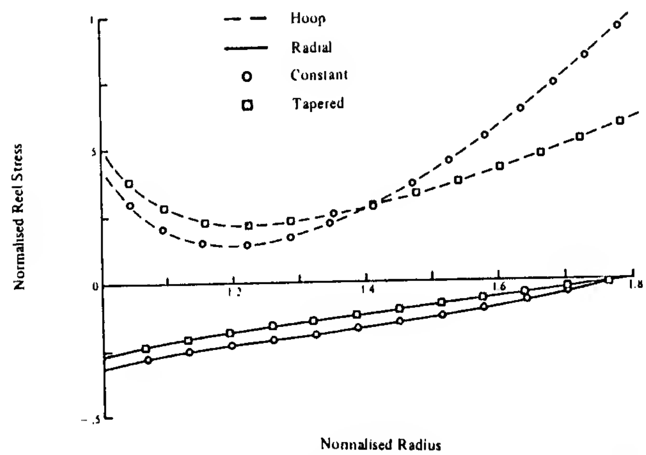

Fig. 7—Predicted effect on the tape's circumferential and radial stresses of a tapered winding tension.

speed has an effect on the amount of air entrapped and thereby changes the tape's radial Young's modulus. The effect of varying the tape's radial Young's modulus is shown in Fig. 8. There, with Eq. [1] holding, we look at the predicted radial and hoop stresses in our basic tape reel as a function of E_r. Note the significant difference between the hoop-stress distribution in the orthotropic tape reel (E_r = 0.17 or 0.345 GPa) as opposed to the isotropic one ($E_r = E_\theta$ = 3.45 GPa). Thus, the assumption of orthotropy leads to significantly different predicted behaviors than that of isotropy.

The radial pressure of the wound tape must be sufficient to avoid slippage in the layers of tape during acceleration. Note from Figs. 5 and 6 that there is little increase in the radial pressure when the stiffness of the hub is increased. Thus, other considerations, such as thermal-stress effects, have to be taken into account to limit the reduction in the radial stress of the wound tape. This is considered in the next section.

ENVIRONMENTAL STRESSES

This section presents a model to predict the stress change induced in a reel of tape when it is subject to a temperature or humidity change from that at which it was wound. The stresses result from the mismatch in the thermal or hygroscopic coefficient of expansion in (a) the hub and the tape, and (b) the tape in the radial and the circumferential directions.

The analysis for a change in humidity is analogous to that for a change in temperature if we use the appropriate coefficients of hygroscopic expansion rather than those of thermal expansion. Therefore, we present only the thermal-stress analysis.

The following undesirable effects of thermal stresses have been noted in the literature (*4*), (*14*):

Caused by a temperature rise:

1. An increased circumferential compression in the inner layers, which may cause wrinkling of the tape near the hub
2. An increased circumferential tension in the outer wraps of tape, which causes increased distortion

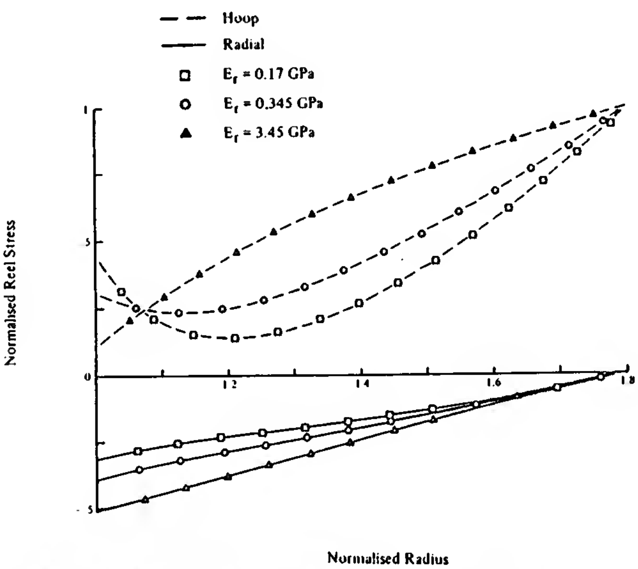

Fig. 8—Predicted effect on the tape's circumferential and radial stresses of the tape's radial modulus.

3. Accelerated stress relaxation; any associated anelastic deformation occurs at a faster rate

Caused by a temperature drop:

A decrease in the radial stress, which increases the chance of interlayer slip on reel acceleration. Also, there is a critical stress at which the radial stress at a radial location can become zero, which has the danger of "layering" (which is a radial separation of the layers) the tape.

Assumptions and Problem Formulation

In addition to the assumptions made in the winding analyses, the following assumptions are made here:

1. The reel of tape behaves as a continuum, which is assumed to be an orthotropic, linearly elastic solid.
2. The materials' properties are independent of temperature and humidity in the range considered here.
3. The temperature (and humidity) change does not vary through the wound tape.

The problem is modeled as two concentric, thin, right-circular cylinders, as shown in Fig. 2. Since neither the tape nor the hub is constrained in the axial direction, there are no thermal stresses in that direction. Because the model is axisymmetric, the in-plane components of the displacement (u_r, u_θ) reduce to

$$u_r = u(r),\ u_\theta = 0 \quad . \qquad [7]$$

The plane-stress, stress-displacement relationship for an orthotropic, linearly thermoelastic solid (*20*) is

$$du/dr - \alpha_r \Delta T = (\sigma_r - \nu_{r\theta}\, \sigma_\theta)/E_\gamma \quad , \qquad [8]$$

$$u/r - \alpha_\theta \Delta T = (\sigma_\theta - \nu_{\theta r}\, \sigma_r)/E_\theta \quad , \qquad [9]$$

where α_r and α_θ are the radial and circumferential coefficients of thermal expansion, respectively, and ΔT is the temperature change.

The stress equation of equilibrium is

$$d\sigma_r/dr + (\sigma_r - \sigma_\theta)/r = 0 \quad . \qquad [10]$$

Boundary Conditions

The boundary conditions are that there are no applied tractions at the inner hub radius, $r = a$, or at the outer radius of the wound tape, $r = c$, while at the interface, $r = b$, the displacement u and the normal stress σ_r are continuous. We write these conditions as

$$\sigma_r^h(a) = 0 \qquad [11]$$

$$\sigma_r^h(b) = \sigma_r^t(b),\ u^h(b) = u^t(b) \qquad [12]$$

$$\sigma_r^t(c) = 0 \qquad [13]$$

where superscripts h and t refer to the hub and tape, respectively. These boundary conditions are more general than those in Refs. (*4*), (*13*), (*14*).

Problem Solution

Substituting for the radial displacement u, using the stress-displacement in Eqs. [8] and [9], in the stress equation of equilibrium of Eq. [10], we get the displacement equation of equilibrium

$$\begin{aligned}(d^2u/dr^2) + (1/r)(du/dr) - (E_\theta/E_r)(u/r^2) \\ = [(\alpha_r + \nu_{\theta r}\alpha_\theta) - (E_\theta/E_r)(\alpha_\theta + \nu_{r\theta}\alpha_r)]\end{aligned} \qquad [14]$$

which governs the radial displacement $u(r)$.

For an isotropic hub

$$E_r = E_\theta \equiv E_h, \nu_{r\theta} = \nu_{\theta r} \equiv \nu_h, \alpha_r = \alpha_\theta \equiv \alpha_h \quad . \qquad [15]$$

So, a solution to the displacement equation of equilibrium in Eq. [14] for the hub is,

$$u^h(r) = C_1^h r + C_2^h/r \quad , (a \leq r \leq b) \quad , \qquad [16]$$

where C_1^h and C_2^h are constants to be determined from the boundary conditions, Eqs. [11]–[13].

For the orthotropic tape, a solution to the displacement equation of equilibrium, Eq. [14] is,

$$u^t(r) = C_1^t r^\beta + C_2^t/r^\beta \quad , (b \leq r \leq c) \quad , \qquad [17]$$

where C_1^t and C_2^t are constants to be determined from the boundary conditions in Eqs. [11]–[13] and $\beta^2 = E_\theta/E_r$.

The case $\beta = 1$ and $\alpha_r^t \neq \alpha_\theta^t$, that is, where magnetic tape is anisotropic only with respect to its thermal expansivity, is not considered since, it does not seem physically realizable.

Considering the boundary conditions in Eqs. [11]–[13] and the stress-displacement relationship in Eqs. [8] and [9], we find that the radial and circumferential components of the thermal stress in the tape are

$$\sigma'_r = [E_r/(1-\nu_{r\theta}\nu_{\theta r})]\,\{(\beta+\nu_{\theta r})\,C_1{}^t r^{\beta-1}$$
$$-(\beta-\nu_{\theta r})(C_2{}^t/r^{\beta+1}) + [(1+\nu_{\theta r})\Omega-\kappa]\Delta T\}$$
$$\sigma'_\theta = [E_\theta/(1-\nu_{r\theta}\nu_{\theta r})]\,\{(1+\beta\,\nu_{r\theta})\,C_1{}^t r^{\beta-1}$$
$$+(1-\beta\,\nu_{r\theta})(C_2{}^t/r^{\beta+1}) + [(1+\nu_{r\theta})\Omega-\lambda\,\kappa]\,\Delta T\}$$

where

$$\kappa = \alpha_r\nu_{\theta r} + \alpha_\theta \quad ; \lambda = (\alpha_\theta+\nu_{r\theta}\alpha_r)/\kappa$$

$$\Omega = [\kappa(1-\lambda\,\beta^2)/(1-\beta^2)] \quad , (\beta \neq 1)$$

and

$$C_1{}^t = [1/(\beta+\nu_{\theta r})][\beta-\nu_\theta)(C_2{}^t/C^{2\beta}) + \gamma^t]$$
$$C_2{}^t = -[(1-\nu_{r\theta}\nu_{\theta r})/(\beta-\nu_{\theta r})]\,\{[\xi_1\xi_5+(E_h/E_r)\xi_3]$$
$$/\ [\xi_1\xi_4-(E_h/E_r)\xi_2]\}\ b^{2\beta+1}$$

where

$$\xi_1 = (1-\nu_h)(b/a)^2 + (1+\nu_h)$$
$$\xi_2 = (1-\nu_{r\theta}\nu_{\theta r})\,\{[1/(\beta+\nu_{\theta r})(b/c)^{2\beta}] + [1/(\beta-\nu_{\theta r})]\}$$
$$\zeta_3 = (\alpha_h - \Omega)\Delta T - [1/(\beta+\nu_{\theta r})](b/c)^{\beta-1}\gamma^t$$
$$\xi_4 = [(b/c)^{2\beta}-1]/[(b/a)^2-1]$$
$$\xi_5 = [1/(1-\nu_{r\theta}\nu_{\theta r})\,\gamma^t]\,\{[(b/a)^{\beta-1} - 1]/[(b/a)^2 - 1]\}$$
$$\gamma^t = [\kappa - (1+\nu_{\theta r})\,\Omega]\,\Delta T$$

Numerical Results

Little information, in general, is known about α_r. In Ref. (*14*), we find that α_r = 50 μm/m/°C.

Looking at the effect of variations in this value, and with

a = 50.8 mm	b = 69.85 mm	c = 125.73 mm
E_θ = 3.45 GPa	$\nu_{\theta r}$ = 0.3	α_θ = 18 μm/m/°C
E_h = 6.89 GPa	ν_h = 0.33	α_h = 27 μm/m/°C

and Eq. [1] holding, we calculate the thermal-stress field induced in wound tape.

We plot in Fig. 9 the change in the hoop stress, due to winding, of a temperature increase of ΔT = 25°C for α_r = 27, 54, and 180 μm/m/°C.

We plot in Fig. 10 the change in the radial stress, due to winding, of a temperature decrease of 25°C for these values of α_r. Also, we show the change in the radial stress, due to winding, of a temperature decrease of 55°C for α_r = 180 μm/m/°C is shown as an extreme case to illustrate the fact that the radial stress can indeed be reduced to zero on cooling. This phenomenon is, of course, dependent on hub material and geometry and, therefore, presents serious consideration on their selection. Storage of tape reels at higher temperatures and humidities that result in significant stress relaxation of the tape would produce the same effect with a lesser hub-tape thermal mismatch or temperature change.

Interlayer slip is the tangential motion of the tape relative to the hub. It is caused by rotational acceleration after exposure of the tape to environmental stress for a period of time. As we have seen, hub stiffness above a certain level has little effect on the radial stress, but any mismatch in the thermal coefficient of expansion between the hub and the tape can have a far greater effect on its value. In extensive testing, we found that a hub with adequate compliance and the least thermal mismatch allowed us to attain the maximum rotational acceleration before tape-to-tape slippage occurred (*25*), (*26*).

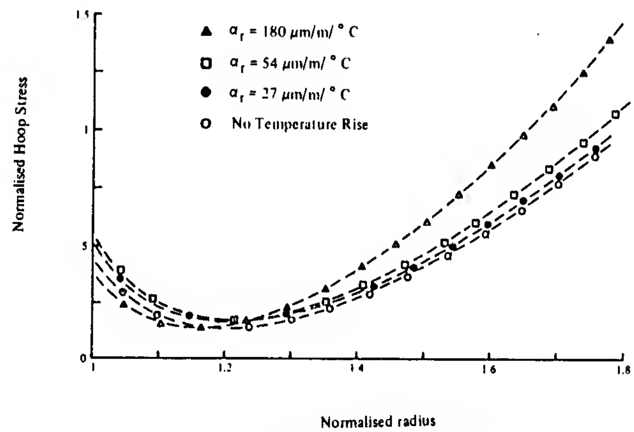

Fig. 9—Predicted effect on the tape's circumferential stress of a temperature rise of 25°C for various tape radial coefficients of thermal expansion.

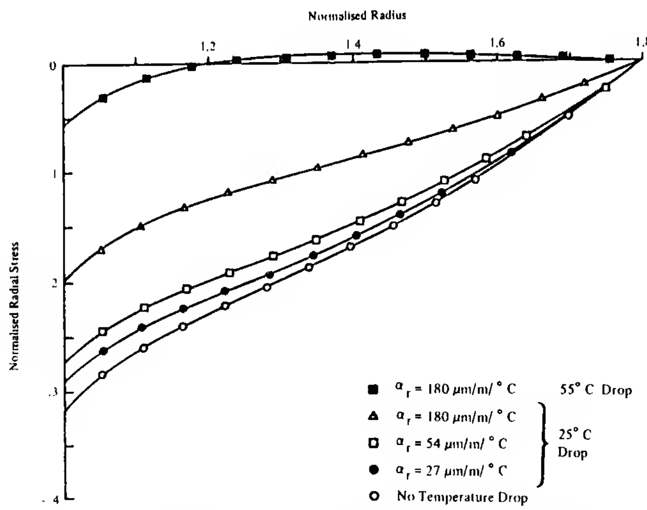

Fig. 10—Predicted effect on the tape's radial stress of a temperature drop of (a) 25°C for various tape radial coefficients of thermal expansion, and (b) 55°C for tape radial coefficient of thermal expansion α_r = 180 μm/m/°C.

SUMMARY

In this paper, we have contrasted the existing analytical techniques to calculate the initial stress field generated during the winding of tape on a hub. We show that while the solution methods differ in ease of use and extendibility, their predictions nonetheless agree. We suggest an approximate technique to measure the radial Young's modulus of a wound tape, which gives the tape's radial modulus as being 1/20th of the tensile modulus of a strip of tape. Using a

pull-tab test, we measured the radial stresses found inside wound tape. These measured data agree very well with the analytical predictions when we restrict the ratio of the tape reel's radial Poisson's ratio to the circumferential Poisson's ratio to be that of the tape reel's radial Young's modulus to the circumferential Young's modulus.

On the basis of this analysis, guidelines for hub design and winding-tension profiles that can prevent the occurrence of commonly known winding defects are given. An analysis of the thermal stress field created in the wound tape by a change in temperature has been described. The lack of a reasonable value for the radial coefficient of thermal expansion for the wound tape allows only a study of the induced thermal-stress field. However, this is sufficient to indicate the application of this analysis to hub geometry and material selection to minimize the adverse effects of thermal stresses on the wound magnetic tape.

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